SAVIJANJE KRUZNE CILINDRICNE
LJUSKE PRI ROTACIONO SIMETRICNOM
OPTERECENJU

-uslovi ravnoteze-

$(1) dM - Tx = 0$

$(2) \frac{Nj}{a} + \frac{dT}{dx} + Z = 0$

$$DJ: \frac{d^4 w}{dx^4} + 4b^4 w = \frac{Z}{K}$$

-deformacija-

$$exz = -\frac{d^2 w}{dx^2} z + \frac{du}{dx}$$

imamo sile: Mx, Tx, Nj, Mj

$Njx = Tj = Nj = Mj$

$$ejz = -\frac{w}{a}$$

$$Mx = -K \frac{d^2 w}{dx^2}; Nj = -Eh \frac{w}{a}$$

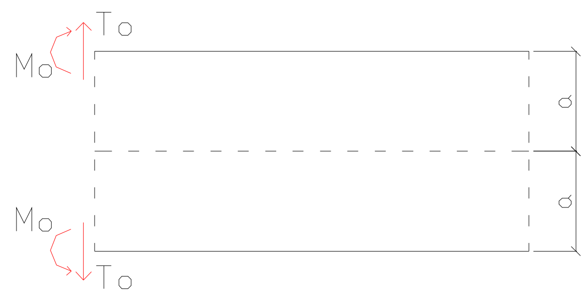
$$Tx = \frac{dMx}{dx}$$

$$w = w_0 + e^{bx}(C1 \cos bx + C2 \sin bx) + e^{-bx}(C3 \cos bx + C4 \sin bx)$$

-duga cilindricna ljuska-($\beta \cdot$l>=5)

$$b = \sqrt[4]{\frac{3(1-u^2)}{a^2 h^a}}$$

$$w = w_0 - \frac{1}{2b^3 K} e^{-bx} [(To + Mob) \cos bx - Mo \sin bx]$$



$$\frac{dw}{dx} = \frac{1}{2b^2 K} e^{-bx} [2bMo \cos bx + To(\cos bx + \sin bx)]$$

$$\frac{d^2 w}{dx^2} = -\frac{1}{bK} e^{-bx} [bMo(\cos bx + \sin bx) + To \sin bx]$$

$$\frac{d^3 w}{dx^3} = \frac{1}{K} e^{-bx} [2bMo \sin bx - To(\cos bx - \sin bx)]$$

$$w_0 = -\frac{Z(x) a^2}{Eh}$$

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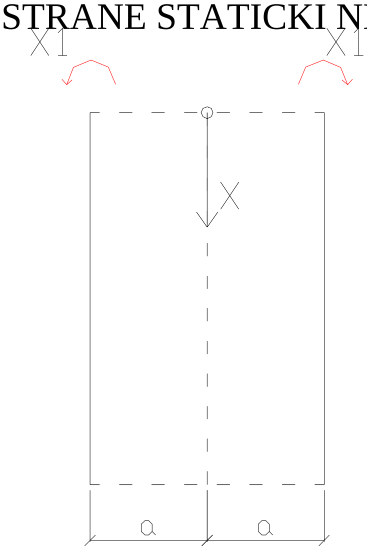
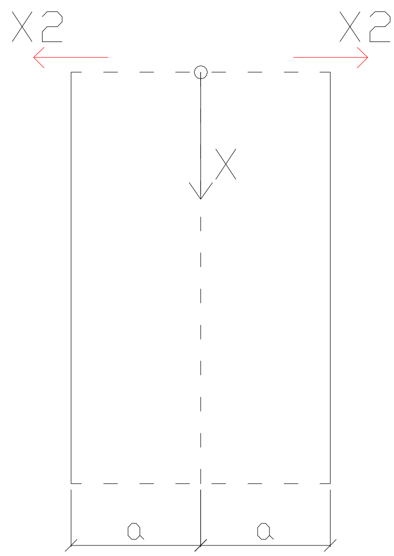
STANJE X1=1(moment):



$$Ed_{11}=E\left(\frac{dw}{dx}\right)_{(x=0)}=E\frac{1}{bK}$$

$$Ed_{21}=Ew_{(x=0)}=E\frac{1}{2b^{2}K}$$

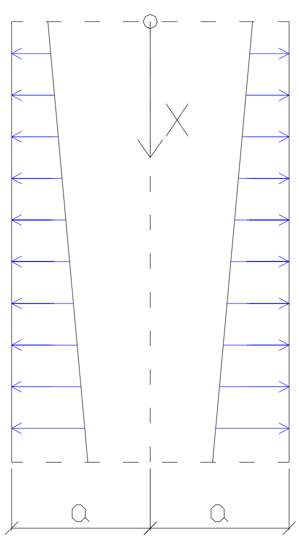
STANJE X2=1(sila):



$$Ed_{22}=Ew_{(x=0)}=E\frac{1}{2b^{3}K}$$

$$Ed_{12}=E\left(\frac{dw}{dx}\right)_{(x=0)}=E\frac{1}{2b^{2}K}$$

STANJE X1=X2=0:
(od „vode“)



$$Z(x)=\ldots$$

$$Nj=-Z\cdot a$$

$$w=a\cdot e_{j}=\frac{a}{Eh}(Nj-u\cdot Nx)(Nx=0uglavnom)$$

$$Ed_{10}=E\cdot\left(\frac{dw}{dx}\right)_{(x=0)}$$

$$Ed_{22}=E\cdot w_{(x=0)}$$

SAVIJANJE KRUZNIH PLOCA

-rotaciono simetricno opterecenje i granicni uslovi:

DJ:

$$\frac{d^4 w}{dr^4}+\frac{2}{r}\frac{d^3 w}{dr^3}-\frac{1}{r^2}\frac{d^2 w}{dr^2}+\frac{1}{r^3}\frac{dw}{dr}=\frac{Z(r)}{K}$$

$w=w_o+w_1$ $\quad$ $w_1=A+B\ln r+Cr^2+Dr^2\ln r$

$w_1=C_1+C_2\ln\rho+C_3\rho^2+C_4\rho^2\ln\rho$ ($\rho=r/a$)

$$M=-\int\frac{dr}{r}\int Z(r)\cdot r\cdot dr$$

$$w_0=-\frac{1}{K}\int\frac{dr}{r}\int M\cdot r\cdot dr - partikularni\,\mathrm{int}\,egral$$

$$Z(r)=Z_0=const\Rightarrow w_0=\frac{Z_0 a^4}{64K}r^4$$

SILE U PRESEKU:

$$Mr=-K\left(\frac{d^2 w}{dr^2}+\frac{u}{r}\frac{dw}{dr}\right)$$

$$Mj=-K\left(\frac{1}{r}\frac{dw}{dr}+u\frac{d^2 w}{dr^2}\right)$$

$$Tr=-K\left(\frac{d^3 w}{dr^3}-\frac{1}{r^2}\frac{dw}{dr}+\frac{1}{r}\frac{d^2 w}{dr^2}\right)$$

-proizvoljno opterecenje:

DJ: $\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}+\frac{1}{r^2}\frac{d^2}{dj^2}\right)\left(\frac{d^2 w}{dr^2}+\frac{1}{r}\frac{dw}{dr}+\frac{1}{r^2}\frac{d^2 w}{dj^2}\right)=\frac{Z(r,j)}{K}$

$w=w_o+w_1$

$w_1=w_o(r)+\sum_{m=0}^{\infty}Wm(r)\cdot\cos(mj)+\sum_{m=1}^{\infty}\overline{W}\,m(r)\cdot\cos(mj)$

$w_o(r)$-izraz za rotaciono simetricno opt(m=0), nista ne zavisi od φ

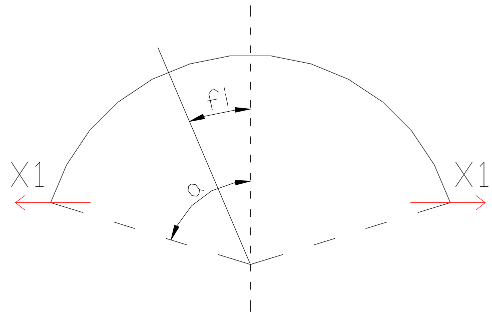
-smena $r=e^t$ =>DJ=>j-na sa konstantnim koeficijentima

m>1: $Wm=Am\cdot r^m+Bm\cdot r^{-m}+Cm\cdot r^{m+2}+Dm\cdot r^{-m+2}$

m=1: $W_1=A_1\cdot r+B_1\cdot r^3+C_1\cdot\frac{1}{r}+D_1\cdot r\ln r$

m=0: $W_0=A_0+B_0\cdot r^2+C_0\cdot\ln r+D_0\cdot r^2\ln r$

SFERNA LJUSKA OPTERECENA
SILAMA PO KONTURI

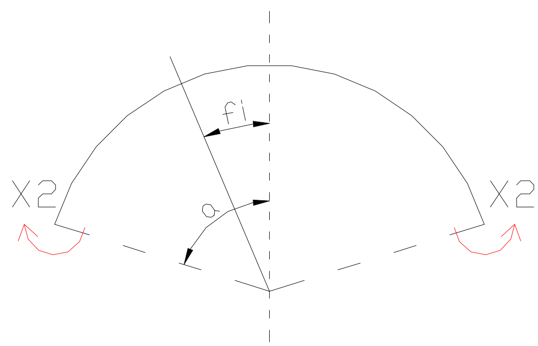


$\Delta R_0(j = a) = 2\frac{gR_0}{Eh}\sin a$ -promena poluprecnika paralelnog kruga

$c(j = a) = -2\frac{g^2 \sin a}{Eh}$ -obrtanje preseka po konturi

$$g = \sqrt{\frac{a}{h}\sqrt{3\left(1-u^2\right)}}$$

SFERNA LJUSKA OPTERECENA
MOMENTIMA PO KONTURI

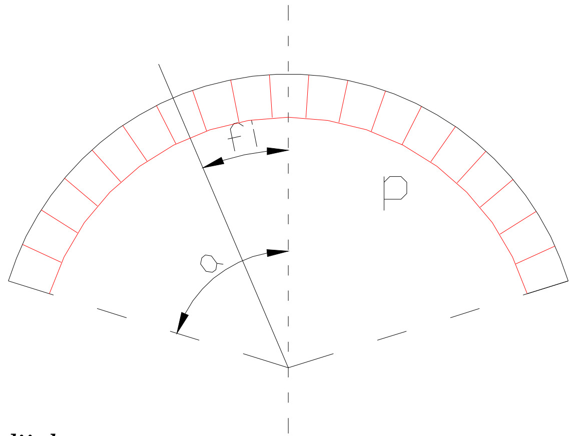


$\Delta R_0(j = a) = 2\frac{g^2 \sin a}{Eh}$ - promena poluprecnika paralelnog kruga

$c(j = a) = -\frac{a}{Kg}$ - obrtanje preseka po konturi

$$K = \frac{Eh^3}{12\left(1-u^2\right)}$$

STANjE: $P \neq 0$



$$Z = \ldots$$

$$Y = \ldots$$

$$P(j) = \int_0^j (\ldots\ldots\ldots)$$

$$N(j) = -\frac{P(j)}{2pa \sin^2 j\, 0}$$

$$(3)\, \frac{Nj}{a} + \frac{Nq}{a} + Z = 0 \Rightarrow Nq$$

$$Ed_{10} = 0 - zaopterecenje\_u\_radijal.pravcu$$

$$Ed_{10} = E \cdot \Delta R_0 = E \cdot e_q \cdot R_0$$

$$e_q = \frac{1}{Eh}\left(N_q - uN_j\right)$$

DIFERENCNI POSTUPAK

-<u>pravougaone ploce</u>-

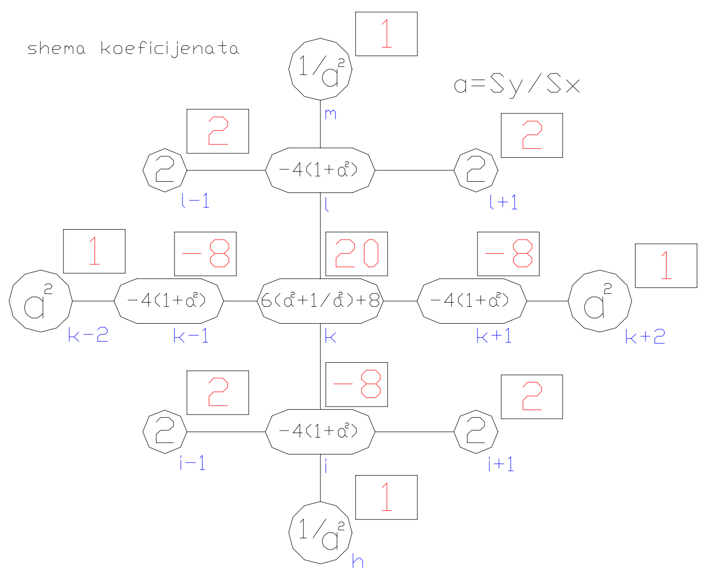
DJ: $\dfrac{d^4w}{dx^4}+2\dfrac{d^4w}{dx^2dy^2}+\dfrac{d^4w}{dy^4}=\dfrac{z(x,y)}{K}$

$$\left(\frac{dw}{dx}\right)_k \approx \frac{w_{k+1}-w_{k-1}}{2Sx}$$

$$\left(\frac{d^2w}{dx^2}\right)_k \approx \frac{w_{k+1}-2w_k+w_{k-1}}{Sx^2}$$

$$\left(\frac{d^3w}{dx^3}\right)_k \approx \frac{w_{k+2}-2w_{k+1}+2w_{k-1}-w_{k-2}}{2Sx^3}$$

$$\left(\frac{d^4w}{dx^4}\right)_k \approx \frac{w_{k+2}-4w_{k+1}+6w_k-4w_{k-1}+w_{k-2}}{Sx^4}$$



$$\left(\frac{dw}{dy}\right)_k \approx \frac{w_l-w_i}{2Sy}$$

$$\left(\frac{d^2w}{dy^2}\right)_k \approx \frac{w_l-2w_k+w_i}{Sy^2}$$

$$\left(\frac{d^3w}{dy^3}\right)_k \approx \frac{w_m-2w_l+2w_i-w_h}{2Sy^3}$$

$$\left(\frac{d^4w}{dy^4}\right)_k \approx \frac{w_m-4w_l+6w_k-4w_i+w_h}{Sy^4}$$

$$\left(\frac{d^2w}{dxdy}\right)_k \approx \frac{w_{l+1}-w_{l-1}-w_{i+1}+w_{i-1}}{4SxSy}$$

$$\left(\frac{d^3w}{dx^2dy}\right)_k \approx \frac{w_{l+1}-2w_l+w_{l-1}-w_{i+1}+2w_i+w_{i-1}}{2Sx^2Sy}$$

$$\left(\frac{d^3w}{dxdy^2}\right)_k \approx \frac{w_{l+1}-2w_{k+1}+w_{i+1}-w_{l-1}+2w_{k-1}-w_{i-1}}{2Sy^2Sx}$$

$$\left(\frac{d^4w}{dx^2dy^2}\right)_k \approx \frac{4w_k-2(w_l+w_i+w_{k+1}+w_{k-1})+w_{l+1}+w_{l-1}+w_{i+1}+w_{i-1}}{Sx^2Sy^2}$$

granicni uslovi

-slobodno oslonjena ivica

$w_{k+2} = -w_k$

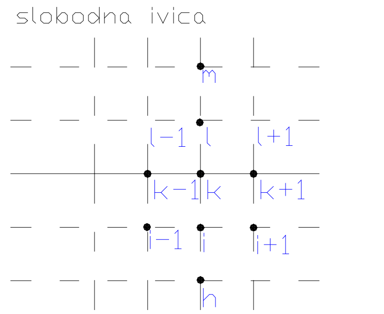
-ukljestenje

$w_{k+2} = w_k$

-slobodna ivica

$My,k = 0....(1)$

$\overline{T}\ y,k = 0....(2)$



$(1) \Rightarrow W_l = 2W_k - W_i - ua^2(W_{k+1} + 2W_k + W_{k-1})$

$(2) \Rightarrow W_m = W_h + [2 + 2a^2(2-u)](W_l - W_i) + a^2(2-u)(W_{i+1} + W_{i-1} + W_{l+1} + W_{l-1})$

-<u>kruzna ploca</u>-

$l m = \frac{s}{r_m}$

-u svakoj tacki ploce u kojoj nam je nepoznat ugib pisemo DJ:

$$(1 + l m)W_{m+2} - \left[2(2 + l_m) + \frac{l_m^2}{2}(2 - l_m)\right] \cdot w_{m+1} + \left(6 + 2l_m^2 + \frac{c}{K}s^4\right)w_m -$$

$$-\left[2(2 - l_m) + \frac{l_m^2}{2}(2 + l_m)\right]w_{m-1+}(1 - l_m) \cdot w_{m-2} = \frac{Zms^4}{K}$$

Zm-vrednost povrsinskog opterecenja u tacki m.

- za r=0:

$$\frac{16}{3}w_2 - \frac{64}{3}w_1 + \left(16 + \frac{c}{K} \cdot s^4\right) \cdot w_0 = \frac{Z_0 s^4}{K}$$

PRESECNE SILE

$$Mr,m = -\frac{K}{S^2}\left[w_{m+1}\left(1 + \frac{u\,l_m}{2}\right) - 2w_m + w_{m-1}\left(1 - \frac{u\,l_m}{2}\right)\right]$$

$$Mj\,,m = -\frac{K}{S^2}\left[w_{m+1}\left(u + \frac{l_m}{2}\right) - 2uw_m + w_{m-1}\left(1 - \frac{l_m}{2}\right)\right]$$

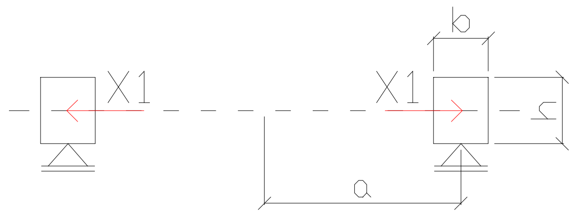
$$Tr,m = -\frac{K}{2S^3}\left[w_{m+2} - w_{m+1}\left(2 - 2l_m + l_m^2\right) - 4l_m w_m + w_{m-1}\left(2 - 2l_m + l_m^2\right) - w_{m-2}\right]$$

KRUZNI PRSTEN

-*RADIJALNA SILA*

-imamo samo normalnu silu N

$$N = a \cdot X_1 \Rightarrow s = \frac{N}{bh} \Rightarrow e = \frac{N}{Ebh} \Rightarrow u = e \cdot a$$



-*RASPODELJENI MOMENTI*

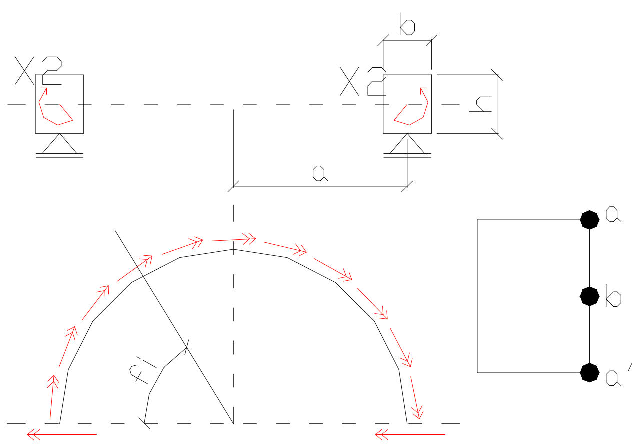
$$M = X_2 \cdot a \Rightarrow s = \frac{M}{I} \cdot y \left(I = \frac{bh^3}{12} \right)$$

$$e = \frac{s}{E} \Rightarrow u = e \cdot a$$

$$d_{11} = u_b^{X1=1}$$

$$d_{22} = c^{X2=1} = -\frac{u_a - u_{a'}}{d}$$

$$d_{12} = u_a^{X2=1}$$



MEMBRANSKA TEORIJA LJUSKI
ROTACIONO SIMETRICNO OPTERECENJE:

$$(1)\frac{dN_q}{dq}R1+\frac{d}{dj}(N_{jq}R0)+N_{jq}R1\cos j+XR0R1=0$$

$$(2)\frac{d}{dj}(N_j R0)+\frac{dN_{jq}}{dq}R1-N_q R1\cos j+YR0R1=0$$ PROIZVOLJNI OPT.

$$(3)\frac{N_j}{R1}+\frac{N_q}{R2}+Z=0$$

-rotaciono simetricno opterecenje-

$X\equiv 0;Y,Z=f(j);N_{jq}=0$

SFERA

$$P(j)=\int_0^j (Z\cos j+Y\sin j)R1\cdot R0\cdot 2p\cdot dj$$

$$Nj=-\frac{P(j)}{2pRo\sin j}$$

$$Nq=-\left(\frac{Nj}{R1}+Z\right)\cdot R2$$

KONUS

$$P(z)=2p\,\frac{tga}{\cos a}\int_0^z (Z\sin a+Y\cos a)\cdot z\cdot dz$$

$$Nj=-\frac{P(z)}{2pRo\cos a}$$

α - ugao u „vrhu“ konusa

$$Nq=-\left(\frac{Nj}{R1}+Z\right)\cdot R2$$

ROTACIONO SIMETRICNA DEFORMACIJA
ROTACIONO SIMETRICNE LJUSKE

$$ej = \frac{1}{Eh}\left(Nj - uNq\right) = \frac{1}{R1}\left(\frac{dv}{dj} - w\right)$$

$$eq = \frac{1}{Eh}\left(Nq - uNj\right) = \frac{dRo}{Ro} = \frac{1}{R2}\left(v \cdot ctgj - w\right)$$

$$c = \frac{1}{R1}\left(v + \frac{dw}{dj}\right)$$

$$v = \left\{\int \frac{1}{Eh}\left[Nj \cdot R1 + uR2 - Nq \cdot R2 + uR1\right]\frac{dj}{\sin j} + C\right\} \cdot \sin j$$

MEMBRANSKA TEORIJA CILINDRICNIH LJUSKI

-uslovi ravnoteze-

$$\left\{\begin{array}{l}(1)\,\dfrac{dNx}{dx}+\dfrac{dNx\,j}{Rd\,j}+X=0(1)\,\dfrac{dNx}{dx}+\dfrac{dNx\,j}{Rd\,j}+X=0\\(2)\,\dfrac{dN\,j}{Rd\,j}+\dfrac{dN\,j\,x}{dx}+Y=0\\(3)\,\dfrac{N\,j}{R}+Z=0\end{array}\right\}$$

$$\left\{\begin{array}{l}N\,j=-Z\cdot R\\N\,j\,x=-\int\left(Y+\dfrac{dN\,j}{Rd\,j}\right)dx+C1(j)\\Nx=-\int\left(X+\dfrac{dN\,j\,x}{Rd\,j}\right)dx+C2(j)\end{array}\right\}$$

$$zaX\equiv 0;Y,Z=f(j)$$

$$\left\{\begin{array}{l}N\,j=-Z\cdot R\\N\,j\,x=-\left(Y+\dfrac{dN\,j}{Rd\,j}\right)\cdot x+C1(j)\\Nx=\dfrac{1}{R}\dfrac{d}{d\,j}\left(Y+\dfrac{dN\,j}{Rd\,j}\right)\dfrac{x^2}{2}-\dfrac{1}{R}\dfrac{dC1(j)}{d\,j}\cdot x+C2(j)\end{array}\right.$$

-kruzna cilindricna ljuska-

R=const=a

-opterecenje razvijamo u red

$$X=\sum_{n=1}^{\infty}Xn\cos n\,j\,;Y=\sum_{n=1}^{\infty}Yn\sin n\,j\,;Z=\sum_{n=1}^{\infty}Zn\cos n\,j\,;$$

-zadrzavamo samo n-te clanove:

$$Nj = -Zn \cdot a$$

$$Njx = -\sin nj \int (Yn + n \cdot Zn) \cdot dx + C1(j)$$

$$Nx = \frac{1}{a} \cos nj \int \left[aXn - n \int (Yn + n \cdot Zn) \cdot dx \right] \cdot dx - \frac{1}{a} \frac{dC1(j)}{dj} + C2(j)$$

-specijalan slucaj-

$$zaXn \equiv 0; Yn, Zn = f(j) \Rightarrow \begin{cases} C1 = A1 \sin nj \\ \hline C2 = A2 \cos nj \end{cases}$$

$$Nj = -Zn \cdot a$$

$$Njx = -[(Yn + n \cdot Zn) \cdot x + A1] \cdot \sin nj$$

$$Nx = \left\{ \frac{n}{a} \left[(Yn + n \cdot Zn) \frac{x^2}{2} - A1 \cdot x \right] + A2 \right\} \cos nj$$

NAPREZANJE U RAVNI U POLARNIM KOORDINATAMA

$$Nr = \frac{1}{r^2}\frac{d^2F}{dj^2} + \frac{1}{r}\frac{dF}{dr};$$

$$N_j = \frac{d^2F}{dr^2};$$

$$N_{rj} = \frac{1}{r^2}\frac{dF}{dj} - \frac{1}{r}\frac{d^2F}{drdj}$$

ROTACIONO SIMETRICNO OPTERECENJE:

$$Nr = \frac{1}{r}\frac{dF}{dr}; Nr = A\frac{1}{r^2} + 2B + C(1 + 2\ln r); e_r = \frac{du}{dr}$$

$$N_j = \frac{d^2F}{dr^2}; N_j = -A\frac{1}{r^2} + 2B + C(3 + 2\ln r); e_j = \frac{u}{r}$$

$$N_{rj} = 0; g_{rj} = 0$$

F=D+Alnr+Br^2+Cr^{2}lnr   D=0(ne utice na naprezanje)

C=0(za kruzni prsten uz uslov kompatibilnosti)

$$e_r = e_j + r\, {de_j}/{dr}$$

A i C=0 za ploce koje 'imaju centar'

Uticaji od temperature:

U=εφ·r $ej = \frac{1}{Eh}(Nj - uNr) + at \cdot t$ ulazi u prelazne uslove ($U^I = U^{II}$)npr.

$$-M = \int_a^b \left[-A \cdot \frac{1}{r^2} + 2B + C(3 + 2\ln r) \right] \cdot rdr = -1\ln\frac{b}{a} + (B + C)(b^2 - a^2) + C(b^2 \ln b - a^2 \ln a)$$

NAVIER-OVO RESENJE

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Amn\cdot\sin(\frac{mpx}{a})\sin(\frac{npy}{b})$$

$$\mathrm{Amn}\,\frac{\mathrm{Zmn}}{\mathrm{K}\pi^{4}\left(\frac{\mathrm{n}^{2}}{\mathrm{b}^{2}}+\frac{\mathrm{m}^{2}}{\mathrm{a}^{2}}\right)^{2}}$$

$$\mathrm{Zmn}:=\frac{4}{\mathrm{a}\cdot\mathrm{b}}\left(\int_{0}^{\mathrm{a}}\int_{0}^{\mathrm{b}}\mathrm{Z}(\mathrm{x},\mathrm{y})\sin\left(\frac{\mathrm{m}\pi\mathrm{x}}{\mathrm{a}}\right)\cdot\sin\left(\frac{\mathrm{n}\pi\mathrm{y}}{\mathrm{b}}\right)\mathrm{dx}\,\mathrm{dy}\right)$$

MORIS-LEVY

$W=W_1+W_0$

$$Yn(y)=\left(An+\frac{npy}{a}Bn\right)ch\frac{npy}{a}+\left(Cn+\frac{npy}{a}Dn\right)sh\frac{npy}{a}$$

$$W_1=\sum_{n=1}^{\infty}Yn(y)\cdot\sin\frac{npx}{a}$$

$$Wo=\sum_{n=1}^{\infty}Won\cdot\sin\frac{npx}{a}$$

$$Won=\frac{Zn\cdot a^4}{Kn^4p^4}$$

$$Zn=\frac{2}{a}\cdot\int_0^a Z(x)\sin\frac{npx}{a}dx$$

$$W(x,y)=W0+W1=$$

$$\sum_{n=1}^{\infty}\left(\frac{Zn\cdot a^4}{Kn^4p^4}+\left(An+\frac{npy}{a}Bn\right)ch\frac{npy}{a}+\left(Cn+\frac{npy}{a}Dn\right)sh\frac{npy}{a}\right)\cdot\sin\frac{npx}{a}$$

- simetricno opterecenje An i Dn
- antimetricno opterecenje Bn i Cn (g.u. Y=+-b/2)

PLOCA OSLONJENA NA GREDU

$$Y = (1)w^{p} = w^{g} \Rightarrow \frac{dWpl}{dx^4} = \frac{dWgr}{dx^4} ////(2)\frac{dWpl}{dy} = jg \Rightarrow \frac{d^2Wpl}{dxdy} = \frac{djg}{dx} = q = \frac{Mt}{GIt}$$

$$(1) \Rightarrow \frac{d^4Wpl}{dx^4} = \frac{\overline{Ty}}{EIg} = \frac{-K}{EIg}\left(\frac{d^3Wpl}{dy^3} + (2-u)\cdot\frac{d^3Wpl}{dx^2dy}\right)$$

$$(2) \Rightarrow \frac{d}{dx}\left(GIt\cdot\frac{d^2Wpl}{dxdy}\right) = \frac{dMt}{dx} = My = -K\left(\frac{d^2Wpl}{dy^2} + u\frac{d^2Wpl}{dx^2}\right)$$

SILE U PRESEKU

$$Mx = -K\left(\frac{d^2W}{dx^2} + u\frac{d^2W}{dy^2}\right)$$

$$My = -K\left(\frac{d^2W}{dy^2} + u\frac{d^2W}{dx^2}\right)$$

$$Mxy = -K(1-u)\frac{d^2W}{dxdy}$$

$$\overline{Tx} = -K\left[\frac{d^3W}{dx^3} + (2-u)\frac{d^3W}{dxdy^2}\right]$$

$$\overline{Ty} = -K\left[\frac{d^3W}{dy^3} + (2-u)\frac{d^3W}{dx^2dy}\right]$$

PLOCE NAPREGNUTE U SVOJOJ RAVNI

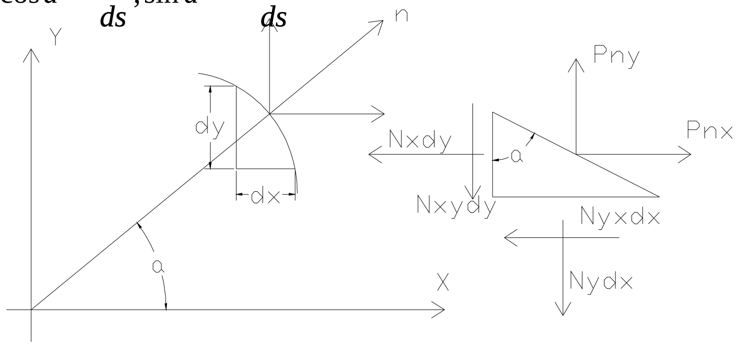
- naponska f-ja F:
- $Nxy = -\frac{d^2F}{dxdy}$; $Nx = \frac{d^2F}{dy^2}$; $Ny = \frac{d^2F}{dx^2}$; ako su X i Y=0

DJ:za plocu opterecenu silama po konturi:

$\frac{d^4F}{dx^4} + 2\cdot\frac{d^4F}{dx^2dy^2} + \frac{d^4F}{dy^4} = 0$ (za plocu opterecenu silama po konturi)

- konturni uslovi:

$\cos a = \frac{dy}{ds}; \sin a = -\frac{dx}{ds}$



$$\frac{dF}{dy} = \int_s p_{nx}ds = Qx$$

$$\frac{dF}{dx} = -\int_s p_{ny}ds = -Qy$$

$$dF = \frac{dF}{dx}dx + \frac{dF}{dy}dy \Rightarrow F = \int_0^s p_{ny}(x - x_s)ds + \int_0^s p_{nx}(y_s - y)ds = M$$

$$\frac{dF}{dn} = \frac{dF}{dx}\cos a + \frac{dF}{dy}(-\sin a)$$

-Naponska f-ja mora da zadovolji DJ.

-opterecenje po konturi na osnovu F(x,y):

$$p_{nx} = Nx\frac{dy}{ds} + Nxy\frac{dx}{ds} \Rightarrow p_{nx} = \frac{d^2F}{dy^2}\cos a + \frac{d^2F}{dxdy}\sin a$$

$$p_{ny} = Ny\frac{dx}{ds} + Nxy\frac{dy}{ds} \Rightarrow p_{ny} = -\frac{d^2F}{dx^2}\sin a - \frac{d^2F}{dxdy}\cos a$$

-SILE U PRESEKU:

$$Nx = D\left(\frac{du}{dx} + u\,\frac{dv}{dy}\right)$$

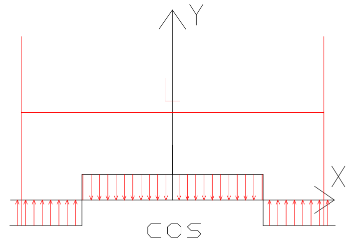
$$Ny = D\left(\frac{dv}{dy} + u\,\frac{du}{dx}\right) \qquad D = \frac{Eh}{\left(1 - u^{2}\right)}$$

$$Nx = \frac{1}{2}\left(1 - u\right)D\left(\frac{du}{dy} + \frac{dv}{dx}\right)$$

POLURAVAN

-opterecenje(periodicno) razvijamo u **sin** ili **cos** red sa periodom L.

-parno opterecenje:



$$p(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{npx}{a}$$

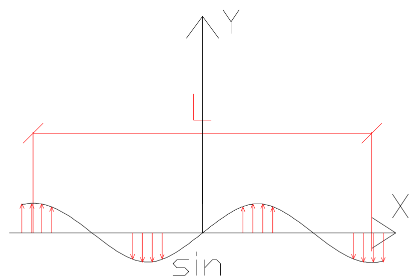
$$a_0 = \frac{4}{L}\int_0^{L/2} p(x)dx$$

$$a_n = \frac{4}{L}\int_0^{L/2} p(x)\cos\frac{npx}{a}dx$$

naponska f-ja:

$$F = \frac{A_o}{2}x^2 + \sum_{n=1}^{\infty} Yn(y)\cos\frac{npx}{a}$$

-neparno opterecenje



$$p(x) = \sum_{n=1}^{\infty} b_n \sin\frac{npx}{a}$$

$$b_n = \frac{4}{L}\int_0^{L/2} p(x)\sin\frac{npx}{a}dx$$

naponska f-ja:

$$F = \sum_{n=1}^{\infty} Yn(y)\sin\frac{npx}{a}$$

$$Yn(y) = \left(An + \frac{npy}{a}Bn\right)e^{-\frac{npy}{a}} + \left(Cn + \frac{npy}{a}Dn\right)e^{\frac{npy}{a}}$$

Cn i Dn=0 za poluravan

GRANICNE USLOVE POSTAVLJAMO PO SILAMA:

$$Nx = \frac{d^2F}{dy^2}; Ny = \frac{d^2F}{dx^2}; Nxy = -\frac{d^2F}{dxdy};$$